1. (13 points) Phineas starts walking at a constant rate on a straight line in the coordinate plane (*i.e.* he exhibits **uniform linear motion**). At time $t = 0$ seconds, Phineas is at the point $(-1, -3)$. At time $t = 10$ seconds, he is at the point $(4, 7)$.

   (a) (5 pts) Find the **parametric equations** for the x and y coordinates of Phineas' location at time t.

   $$x = a + bt, \quad y = c + dt$$

   $t = 0, \, x = -1 \Rightarrow -1 = a + b(0) \Rightarrow a = -1$

   $t = 10, \, x = 4 \Rightarrow 4 = -1 + b(10) \Rightarrow b = \frac{5}{10} = \frac{1}{2}$

   $$\boxed{x = -1 + \tfrac{1}{2}t}$$

   $t = 0, \, y = -3 \Rightarrow -3 = c + d(0) \Rightarrow c = -3$

   $t = 10, \, y = 7 \Rightarrow 7 = -3 + d(10) \Rightarrow d = 1$

   $$\boxed{y = -3 + t}$$
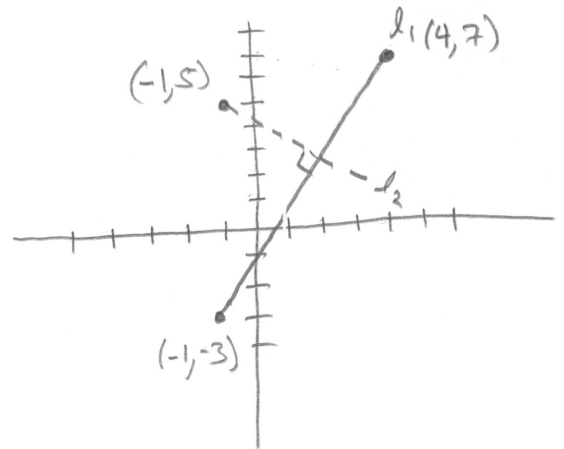
   (b) (8 pts) Ferb is standing still in the coordinate plane at the point $(-1, 5)$. At what time, t, is Phineas closest to Ferb?

   $\ell_1: \quad m = \dfrac{7 - {-3}}{4 - {-1}} = \dfrac{10}{5} = 2$

   $y = 2(x - {-1}) + {-3}$

   $y = 2x + 2 - 3$

   $$\boxed{y = 2x - 1}$$

   $\ell_2: \quad m = -\dfrac{1}{2}, \quad y = -\dfrac{1}{2}(x - {-1}) + 5$

   $$\boxed{y = -\tfrac{1}{2}x - \tfrac{1}{2} + 5 = -\tfrac{1}{2}x + \tfrac{9}{2} = -0.5x + 4.5}$$

   INTERSECT: $2x - 1 = -\tfrac{1}{2}x + \tfrac{9}{2}$

   $\tfrac{5}{2}x = \tfrac{11}{2}$

   $$\boxed{x = \tfrac{11}{5} = 2.2}$$

   NOTE: $y = 2x - 1 = \tfrac{22}{5} - 1 = \tfrac{17}{5} = 3.4$

   TIME: $x = \tfrac{11}{5} \Rightarrow$

   $\tfrac{11}{5} = -1 + \tfrac{1}{2}t$

   $\tfrac{16}{5} = \tfrac{1}{2}t$

   $$\boxed{t = \tfrac{32}{5} = 6.4 \text{ seconds}}$$

7. Chloë and and Joë are walking around the coördinate plane. They both begin walking at the same time, in straight lines at constant speeds.

(a) **[3 points]** Chloë starts at $(-2, -3)$ and walks east at a speed of 4 units per second.

Give parametric equations for Chloë's coördinates after t seconds.

$$x = -2 + 4t$$

$$y = -3$$

(b) **[4 points]** Joë begins at the point $(6, 3)$ and walks towards the point $(14, -5)$, reaching it in 4 seconds.

Give parametric equations for Joë's coördinates after t seconds.

$x_0 = 6$ $\qquad$ $y_0 = 3$

$x_1 = 14$ $\qquad$ $y_1 = -5$

$\Delta x = 8$ $\qquad$ $\Delta y = -8$

$\Delta t = 4$

$x = 6 + \frac{8}{4}t$

$y = 3 + \frac{-8}{4}t$

$$x = 6 + 2t$$

$$y = 3 - 2t$$

(c) **[5 points]** When are Chloë and Joë closest together?

$$dist = \sqrt{\left((-2+4t)-(6+2t)\right)^2 + \left(-3-(3-2t)\right)^2}$$

$$= \sqrt{\left(-8+2t\right)^2 + \left(-6+2t\right)^2}$$

$$= \sqrt{64-32t+4t^2 +36-24t +4t^2}$$

$$= \sqrt{8t^2-56t +100} \quad \leftarrow \text{quadratic!}$$

$$\text{min at } h = \frac{-b}{2a} = \frac{56}{16} = \boxed{3.5 \text{ seconds}}$$